## **Chapter 7 Homework**

7-5. The ionic strength 0.030 M is halfway between the values 0.01 and 0.05 M. Therefore, the activity coefficient will be halfway between the tabulated values:  $\gamma = \frac{1}{2}(0.914 + 0.86) = 0.887.$ 

7-6. (a) 
$$\log \gamma = \frac{-0.51 \cdot 2^2 \cdot \sqrt{0.083}}{1 + (600\sqrt{0.083} / 305)} = -0.375 \Rightarrow \gamma = 10^{-0.375} = 0.42_2$$
  
(b)  $\gamma = \left(\frac{0.083 - 0.05}{0.1 - 0.05}\right) (0.405 - 0.485) + 0.485 = 0.43_2$ 

**7-9.** The solubility of Hg<sub>2</sub>Br<sub>2</sub> is small, so we assume that Hg<sub>2</sub>Br<sub>2</sub> contributes negligible Br<sup>-</sup> to 0.001 00 M KBr.

$$\mu = 0.001 \ 00 \ \text{M}, \ [\text{Br}] = 0.001 \ 00 \ \text{M}, \ \gamma_{\text{Hg}^{2+}} = 0.867, \ \gamma_{\text{Br}} = 0.964$$

$$K_{\rm sp} = 5.6 \times 10^{-23} = [\text{Hg}_2^{2+}] \gamma_{\text{Hg}2+} [\text{Br}^{-}]^2 \gamma_{\text{Br}}^2$$
$$= [\text{Hg}_2^{2+}] (0.867) (0.001\ 00)^2 (0.964)^2 \implies [\text{Hg}_2^{2+}] = 7.0 \times 10^{-17} \text{ M}$$

Check our assumption: Yes, Br<sup>-</sup> from Hg<sub>2</sub>Br<sub>2</sub> is negligible.

**7-10.** The solubility of Ba(IO<sub>3</sub>)<sub>2</sub> is small, so we assume that Ba(IO<sub>3</sub>)<sub>2</sub> contributes negligible IO<sub>3</sub><sup>-</sup> to 0.100 M (CH<sub>3</sub>)<sub>4</sub>NIO<sub>3</sub>.

$$\mu = 0.100 \text{ M}, \text{ [IO_3^-]} = 0.100 \text{ M}, \gamma_{\text{Ba}^{2+}} = 0.38, \gamma_{\text{IO}_3^-} = 0.775$$
$$K_{\text{sp}} = 1.5 \times 10^{-9} = \text{ [Ba^{2+}]} \gamma_{\text{Ba}^{2+}} \text{ [IO_3^-]}^2 \gamma_{\text{IO}_3^-}$$
$$= \text{ [Ba^{2+}]} (0.38) (0.100)^2 (0.775)^2 \implies \text{ [Ba^{2+}]} = 6.6 \times 10^{-7} \text{ M}$$

7-12. Ionic strength = 0.010 M from NaOH + 0.012 M from LiNO<sub>3</sub> = 0.022 M. Interpolating in Table 7-1 gives  $\gamma_{OH}$  = 0.873.

$$[H^+]\gamma_{H^+} = \frac{K_w}{[OH^-]\gamma_{OH^-}} = \frac{1.0 \times 10^{-14}}{(0.010)(0.873)} = 1.15 \times 10^{-12}$$
  
pH = -log(1.15 × 10^{-12}) = 11.94

If we had neglected activities, pH  $\approx -\log[H^+] = -\log\frac{K_W}{[OH^-]} = 12.00$ 

7-18. 
$$[H^+] + 2 [Ca^{2+}] + [Ca(HCO_3)^+] + [Ca(OH)^+] + [K^+] = [OH^-] + [HCO_3^-] + 2[CO_3^{2-}] + [ClO_4^-]$$

7-21. (a) Charge balance:  $2[Mg^{2+}] + [H^+] + [MgBr^+] + [MgOH^+] = [Br^-] + [OH^-]$ Mass balance: total Br = 2(total Mg)  $[MgBr^+] + [Br^-] = 2\{[Mg^{2+}] + [MgBr^+] + [MgOH^+]\}$ 

- (b)  $[Mg^{2+}] + [MgBr^+] + [MgOH^+] = 0.2 M$  $[MgBr^+] + [Br^-] = 0.4 M$
- **7-23.**  $[CH_3CO_2^-] + [CH_3CO_2H] = 0.1 M$

7-25. 3 (total Fe) = 2 (total sulfur)  

$$3\{[Fe^{3+}] + [Fe(OH)^{2+}] + [Fe(OH)^{+}_{2}] + 2[Fe_{2}(OH)^{4+}_{2}] + [FeSO^{+}_{4}]\}$$
  
 $= 2\{[FeSO^{+}_{4}] + [SO^{2-}_{4}] + [HSO^{-}_{4}]\}$ 

We write 2 in front of  $[Fe_2(OH)_2^{4+}]$  because  $Fe_2(OH)_2^{4+}$  contains 2 Fe.

7-26. (a) Pertinent reactions:

$$A^{-} + H_2O \rightleftharpoons HA + OH^{-}$$
  $K_b = \frac{[HA]\gamma_{HA}[OH^{-}]\gamma_{OH^{-}}}{[A^{-}]\gamma_{A^{-}}} = 5.7 \times 10^{-10}$  (A)

$$H_2O \rightleftharpoons^{K_w} H^+ + OH^- \qquad K_w = [H^+]\gamma_{H^+} [OH^-]\gamma_{OH^-} = 1.0 \times 10^{-14}$$
(B)

Charge balance: 
$$[H^+] + [Na^+] = [OH^-] + [A^-]$$
 (C)

Mass balance:  $[Na^+] = 0.01 M \equiv F$  (D)

Mass balance:  $[HA] + [A^{-}] = 0.01 M \equiv F$  (E)

(b) Now we neglect activity coefficients. We will make the following substitutions in the charge balance:

$$[OH^-] = K_w/[H^+]$$
From Equation A: 
$$[HA] = \frac{K_b[A^-]}{[OH^-]}$$
(F)

From Equation E:  $[HA] = F - [A^-]$  (G)

Now equate the expressions for [HA] from Equations F and G to solve for [A<sup>-</sup>]  $\frac{K_b[A^-]}{[OH^-]} = F - [A^-]$ 

$$[A^{-}]\left(\frac{K_{b}}{[OH^{-}]}+1\right) = F \implies [A^{-}] = \frac{F[OH^{-}]}{K_{b}+[OH^{-}]}$$

Now substitute  $K_w/[H^+]$  for each [OH<sup>-</sup>] in the equation above to get

$$[A^{-}] = \frac{FK_{w}/[H^{+}]}{K_{b} + K_{w}/[H^{+}]} = \frac{FK_{w}}{K_{b}[H^{+}] + K_{w}}$$
(H)

We can now substitute for all terms in the charge balance using [A<sup>-</sup>] from Equation H, [OH<sup>-</sup>] =  $K_w/[H^+]$ , and [Na<sup>+</sup>] = F: Charge balance: [H<sup>+</sup>] + [Na<sup>+</sup>] = [OH<sup>-</sup>] + [A<sup>-</sup>]

$$[H^+] + F = \frac{K_w}{[H^+]} + \frac{FK_w}{K_b[H^+] + K_w}$$
(I)

Equation I has the form we were looking for. The only unknown is  $[H^+]$ . For convenience in finding a numerical solution, I will rearrange as follows:

$$0 = \frac{K_{\rm w}}{[{\rm H}^+]} + \frac{{\rm F}K_{\rm w}}{K_{\rm b}[{\rm H}^+] + K_{\rm w}} - [{\rm H}^+] - {\rm F}$$
(J)

The following spreadsheet evaluates the right side of Equation J in cell D9. Guess a value for  $[H^+]$  in cell B7. Before using Goal Seek in Excel 2007, click the Microsoft Office button at the top left of the spreadsheet, click on Excel Options, and then on Formulas. Set Maximum Change to 1e-14. In earlier versions of Excel, go to Tools and Options and select the Calculations tab and set Maximum change to 1e-14. Then execute Goal Seek to vary  $[H^+]$  in cell B7 until the sum in cell D9 is close to 0 (within 1e-14). The answer in cell B7 is  $[H^+] = 4.19 \times 10^{-9}$  M or pH = 8.38.

Cells C15:C17 compute other concentrations from the relations

$[OH^-] = \frac{K_w}{[H^+]}$	$[\mathbf{A}^{-}] = \frac{\mathbf{F}K_{\mathbf{w}}}{K_{\mathbf{b}}[\mathbf{H}^{+}] + K_{\mathbf{w}}}$	$[HA] = \frac{K_b[A]}{[OH]}$

	A	В	С	D	E	
1	Finding the species in NaOAc solution					
2	Using GOAL SEEK to Solve for [H <sup>+</sup> ]					
3	K <sub>w</sub> =	1.00E-14		1		
4	K <sub>b</sub> =	5.70E-10		-		
5	F =	0.01				
6						
7	Guess for $[H^+] =$	4.19E-09		pH = -log[H <sup>+</sup> ] =	8.38	
8						
9	$K_w/[H^+] + FK_w/(K_b$	[H <sup>+</sup> ] + K <sub>w</sub> ) -[	H <sup>+</sup> ] - F =	1.41E-16		
10		1				
11	Tools> Options> Calculation> Maximum change = 1e-14					
12	Use GOAL SEEK to vary [H <sup>+</sup> ] in cell B7 until the sum in					
13	cell D9 is equ	ual to zero				
14						
15		= K <sub>w</sub> /[H⁺} =	2.39E-06			
16	$[A^{-}] = FK_w/(K_b[H^{+}])$	+ K <sub>w</sub> ) =	1.00E-02			
17	[HA] = K <sub>b</sub>	[A]/[OH <sup>-</sup> ] =	2.39E-06			

7-27. (a) Pertinent reactions:

$$Ca(OH)_{2}(s) \stackrel{K_{sp}}{\rightleftharpoons} Ca^{2+} + 2OH^{-} \qquad K_{sp} = [Ca^{2+}]\gamma_{Ca}^{2+}[OH^{-}]^{2}\gamma_{OH^{-}}^{2} = 6.5 \times 10^{-6}$$

$$Ca^{2+} + OH^{-} \stackrel{K_{1}}{\rightleftharpoons} CaOH^{+} \qquad K_{1} = \frac{[CaOH^{+}]\gamma_{CaOH^{+}}}{[Ca^{2+}]\gamma_{Ca}^{2+}[OH^{-}]\gamma_{OH^{-}}} = 2.0 \times 10^{1}$$

$$H_{2}O \stackrel{K_{w}}{\rightleftharpoons} H^{+} + OH^{-} \qquad K_{w} = [H^{+}]\gamma_{H^{+}} [OH^{-}]\gamma_{OH^{-}} = 1.0 \times 10^{-14}$$

$$Charge \text{ balance: } 2[Ca^{2+}] + [CaOH^{+}] + [H^{+}] = [OH^{-}]$$

$$Mass \text{ balance: } [OH^{-}] + [CaOH^{+}] = 2\{[Ca^{2+}] + [CaOH^{+}]\} + [H^{+}]$$

$$species \text{ containing } OH^{-} \qquad species \text{ containing } Ca^{2+}$$

(Mass balance gives the same result as charge balance.)

There are 4 equations (3 equilibria and charge balance) and 4 unknowns:

[Ca<sup>2+</sup>], [CaOH<sup>+</sup>], [H<sup>+</sup>], and [OH<sup>-</sup>].

(b) The approximations we make are to disregard the activity coefficients and to neglect [H<sup>+</sup>] in the charge balance because [H<sup>+</sup>] << [OH<sup>-</sup>] in basic solution. The charge balance becomes

 $2[Ca^{2+}] + [CaOH^+] = [OH^-]$ (A) Substituting [CaOH^+] =  $K_1[Ca^{2+}][OH^-]$  into (A) gives  $2[Ca^{2+}] + K_1[Ca^{2+}][OH^-] = [OH^-] \implies [Ca^{2+}] = \frac{[OH^-]}{2 + K_1[OH^-]}$ Substitute this expression for  $[Ca^{2+}]$  into the solubility product:  $K_{sp} = [Ca^{2+}][OH^-]^2 = \frac{[OH^-]^3}{2 + K_1[OH^-]}$ (B)

	A	В	С	D
1	Ca(OH) <sub>2</sub> solubility			
2				
3	K <sub>sp</sub> =		[OH <sup>-</sup> ] <sub>guess =</sub>	[OH <sup>-</sup> ] <sup>3</sup> /(2 + K <sub>1</sub> [OH <sup>-</sup> ]) =
4	6.5E-06		0.0253528	6.5000E-06
5	K <sub>1</sub> =			
6	2.0E+01		[Ca <sup>2+</sup> ] =	[CaOH <sup>⁺</sup> ] =
7			0.0101126	0.0051276
8				
9	$D4 = C4^{3}/(2 + A6^{*}C4)$			[H <sup>+</sup> ] = K <sub>w</sub> /[OH] =
10	C7 = A4/C4^2			3.94E-13
11	D7 =A6*C7*C4			

We solve Equation B in the spreadsheet by guessing [OH<sup>-</sup>] in cell C4 until the expression in cell D4 is equal to  $K_{sp}$ . We used Goal Seek for this purpose.

Results:  $[Ca^{2+}] = 0.010 \text{ 1 M}$ [OH-] = 0.025 4 M  $[CaOH^+] = 0.005 \ 1 M$  $[H^+] = K_w / [OH^-] = 3.9 \times 10^{-13} M$ 

Total dissolved Ca =  $0.010 \ 1 + 0.005 \ 1 = 0.015 \ 2 \ M$ The formula mass of Ca(OH)<sub>2</sub> is 74.09 g/mol, so  $0.015 \ 2 \ M$  is  $1.1_3 \ g/L$ . The *Handbook of Chemistry and Physics* lists the solubility of Ca(OH)<sub>2</sub> as  $1.85 \ g/L$  at 0°C and 0.77 g/L at 100°C.