

Complete five (5) of the following problems. CLEARLY mark the problems you do not want graded. You must show your work to receive credit for problems requiring math. Report your answers with the appropriate number of significant figures.

Do five of problems 1-7. Clearly mark the problems you do not want graded. (16 pts each)

1. A solution was prepared by dissolving 1.795 grams of a solid sample containing an unknown amount of lead in a total of 100.00 mL of solution, which was labeled solution A. Before analysis, 5.00 mL of solution A was pipetted into a 100.00 mL volumetric flask, mixed and diluted to the mark to form solution B. Then 10.00 mL of solution B was pipetted into a 25.00 mL volumetric flask, mixed and diluted to the mark to make solution C. Analysis of solution C determined that it had a lead concentration of 11.2 ppm. What was the percent lead by mass in the original solid sample? You may assume a density of 1.00 g/mL for all solutions.

It is useful to think of ppm Pb as $\mu\text{g Pb/mL}$ solution (or mg Pb/L solution). This isn't essential, but it makes the dimensional analysis a little more streamlined. There are several approaches to the answer, here is an example of one approach.

We need to first account for each of the dilutions to determine the concentration of mercury in the original solution:

$$\frac{11.2 \mu\text{g Pb}}{\text{mL C}} \times \frac{25.00 \text{ mL C}}{10.00 \text{ mL B}} \times \frac{100.00 \text{ mL B}}{5.00 \text{ mL A}} = \frac{560 \mu\text{g Pb}}{\text{mL A}}$$

Now we can determine the mass of Pb in solution A:

$$\frac{560 \mu\text{g Pb}}{\text{mL A}} \times 100.00 \text{ mL A} \times \frac{10^{-6} \text{ g Pb}}{\mu\text{g Pb}} = 0.0560 \text{ g Pb}$$

Finally, determine %Pb:

$$\frac{0.0560 \text{ g Pb}}{1.795 \text{ g sample}} \times 100 \% = \mathbf{3.12 \% Pb}$$

2. A Standard Reference Material is certified to contain 45.4 ppm of an organic contaminant in soil. You analyze this material to characterize a new method you are developing. Your analysis gives values of 47.8, 47.4, 45.3, 48.1, and 47.2 ppm. Evaluate the results for suspect data and determine whether your results indicate the presence of systematic error in your method at the 95% confidence level. Justify your answer.

Based on the full dataset, the mean is 47.2 ppm, and $s = 1.1$ ppm

With all of the other data bunched around 47 and 48 ppm, the point at 45.3 ppm should look a little odd and worthy of a Q-test. Q for 5 observations is 0.64

$$\frac{47.2-45.3}{48.1-45.3} = \frac{1.9}{2.8} = \mathbf{0.68 \text{ is } > \mathbf{0.64 \text{ so omit 45.3}}$$

If you choose to do the Grubb's test, G for 5 observations is 1.672:

$$\frac{47.2-45.3}{1.1} = \frac{1.9}{1.1} = \mathbf{1.72 > 1.672 \text{ so omit 45.3}}$$

Once the outlier is omitted, the average becomes 47.6 ppm with a standard deviation of 0.4 ppm. To determine whether systematic error is indicated, determine if the "true value" falls within the confidence interval. (using the 95% confidence level). For 3 degrees of freedom and 95%, $t_{\text{table}} = 3.182$

$$\text{CI} = 47.6 \pm \frac{ts}{\sqrt{n}} = 47.6 \pm \frac{3.182 \times 0.4}{\sqrt{4}} = 47.6 \pm 0.64$$

So, the confidence range is 47.6 ± 0.6 ppm, which does not include the true value, therefore, there seems to be an indication of systematic error (at least a 5% chance).

You could also calculate a t value to compare to the tabulated t:

$$t_{\text{calc}} = \frac{|\text{true value} - \bar{x}|}{s} \sqrt{n} = \frac{|45.4 - 47.6|}{0.4} \sqrt{4} = 11$$

Since $t_{\text{calc}} > t_{\text{table}}$ there is a statistically significant difference.

3. Acid solutions can be standardized using primary standard sodium carbonate, much like base solutions can be standardized using pure KHP as we did in lab. Below is data from a titration of a sodium carbonate sample with a solution of hydrochloric acid of unknown concentration. In this titration, approximately 25 mL of distilled water was used to dissolve the sodium carbonate that was dispensed from the weighing bottle into an Erlenmeyer flask. What is the molarity of the hydrochloric acid solution with its absolute uncertainty?

Initial mass of weighing bottle and sodium carbonate	32.1834±0.0002 g
Final mass of weighing bottle after sample was removed	30.9651±0.0002 g
Initial buret reading	2.83±0.02 mL
Final buret reading	39.45±0.02 mL
Molar mass of sodium carbonate	105.9885±0.0002 g/mol

Our reaction of interest is:



Our general calculation is:

$$(\text{m} \pm \text{e}_m) \text{g Na}_2\text{CO}_3 \times \frac{1 \text{ mol Na}_2\text{CO}_3}{(\text{MM} \pm \text{e}_{\text{MM}}) \text{g Na}_2\text{CO}_3} \times \frac{2 \text{ mol HCl}}{1 \text{ mol Na}_2\text{CO}_3} \times \frac{1}{(\text{V} \pm \text{e}_v) \text{L solution}} = [\text{HCl}]$$

We are given the molar mass and its uncertainty, but need to calculate the mass and uncertainty of Na_2CO_3 and volume and uncertainty of HCl solution.

$$\text{Mass Na}_2\text{CO}_3 = 32.1834 - 30.9651 \text{ g} = 1.2183 \text{ g}$$

Uncertainty in mass:

$$e_m = \sqrt{(0.0002 \text{ g})^2 + (0.0002 \text{ g})^2} = 0.0002_8 \text{ g}$$

$$\text{Volume HCl} = 39.45 - 2.83 \text{ mL} = 36.62 \text{ mL}$$

Uncertainty in volume:

$$e_v = \sqrt{(0.02 \text{ mL})^2 + (0.02 \text{ mL})^2} = 0.02_8 \text{ mL}$$

Now we can insert these values into our calculation

$$\frac{(1.2183 \pm 0.0002_8) \text{ g}}{\text{Na}_2\text{CO}_3} \times \frac{1 \text{ mol Na}_2\text{CO}_3}{(105.9885 \pm 0.0002) \text{ g Na}_2\text{CO}_3} \times \frac{2 \text{ mol HCl}}{1 \text{ mol Na}_2\text{CO}_3} \times \frac{1}{(37.16 \pm 0.02_8) \times 10^{-3} \text{ L solution}} = (0.62778 \pm e_M) \text{ M HCl}$$

$$e_M = 0.62778 \sqrt{\left(\frac{0.0002_8 \text{ g}}{1.2183 \text{ g}}\right)^2 + \left(\frac{0.02_8 \text{ mL}}{36.62 \text{ mL}}\right)^2 + \left(\frac{0.0002 \text{ g/mol}}{105.9885 \text{ g/mol}}\right)^2} = (0.061866 \text{ M})(0.00081) = 0.000406 \text{ M}$$

So, the HCl concentration is $0.6278 \pm 0.0005 \text{ M}$

4. Complete both parts in a few sentences. (8 pts each part)
- a. Why do systematic (determinate) errors typically have a larger impact on the accuracy of a measurement than random (indeterminate) errors?

By their nature, systematic errors (such as miscalibrated equipment), result in the experimentally determined value being offset from the true value by a constant amount. For example, a poorly calibrated volumetric pipet may deliver an extra 0.10 mL of solution, but it will reproducibly deliver this erroneous volume. Therefore every independent measurement will be skewed by the same amount, leading to poor accuracy.

Indeterminate (or random) errors involve both positive and negative deviations from the true value. While they may vary in size, the scatter is always around the true value. Therefore, as long as you collect a reasonable number of data points, the average should be close to the true value (good accuracy), although reproducibility may be poor (poor precision).

- b. You have been tasked with determining the limit of detection for a new instrumental technique for the determination of lead in drinking water. Describe how you would accomplish this task. Include a general description of the samples you would make and measure and how these measurements would be used to determine the LOD.

To find the LOD, you need to know the precision of the measurement and how the measurement response is related to concentration. Remember, LOD must have units corresponding to amount of analyte.

Your process should include the following.

- You must prepare a series of solutions of known concentration, including a blank.
- You must measure the response of each of these solutions
- You must measure one of the solutions multiple times to determine the standard deviation of the response.
- You determine the signal at the detection limit by: $I_{LOD} = I_{blank} + 3S_{blank}$ where I_{blank} is the average signal of a blank measurement and S_{blank} is the standard deviation of the measurement.
- You determine the relationship between signal and concentration by plotting a calibration curve and fitting it (usually with a line)
- You then use the equation for the fit to convert I_{LOD} to concentration units.

5. You are working to develop a new method for the determination of the sulfur content in coal. If successful, your method has the potential to be very valuable. To validate your method, you decide to compare it to an established, “Industry Standard” method. The weight percent sulfur of four different coal samples (each containing different amounts of S) was measured by the two different methods. Does your method give results that are consistent with the Industry Standard at the 95% confidence level?

Sample	1	2	3	4
Industry Standard Method	1.157	1.538	1.795	2.284
Your Method	1.151	1.534	1.785	2.280

Since these values are for single measurements of multiple samples, we have to base our decision on the differences between the results for each sample. First we need to calculate an s_d :

Sample	1	2	3	4
Industry Standard Method	1.157	1.538	1.795	2.284
Your Method	1.151	1.534	1.785	2.280
d	0.006	0.004	0.010	0.004
Average d	0.006			
$(d - d_{\text{average}})^2$	0	$(0.002)^2$	$(0.004)^2$	$(0.002)^2$

$$s_d = \sqrt{\frac{(0)^2 + (0.002)^2 + (0.004)^2 + (0.002)^2}{4 - 1}} = 0.00283$$

Now we can calculate a t value:

$$t_{\text{calculated}} = \frac{\bar{d}}{s_d} \sqrt{n} = \frac{0.006}{0.00283} \sqrt{4} = 4.243$$

The critical value of t for 3 degrees of freedom is 3.182.

Since $t_{\text{calculated}} > t_{\text{critical}}$ there is a statistically significant difference between the two methods.

Since these are the results for individual measurements of different samples, it is not appropriate to use s_{pooled} which compares replicate results of a single sample on two methods.

6. You are working to determine the concentration of acetaminophen in an analgesic preparation by absorbance spectrophotometry. You prepare an unknown solution and series of standard solutions and measure the absorbance of each solution at 255 nm. The resulting data is shown below. Assuming a linear relationship between absorbance and concentration, describe how you would determine the 95% confidence interval for the acetaminophen concentration of the unknown. You DO NOT need to do any calculations, just clearly describe how you would go from the raw data to find the 95% confidence interval for the unknown. What key parameters will you calculate along the way? What value do you select for t ?

[acetaminophen] (mM)	Absorbance at 255 nm
0.00	0.279
10.37	0.602
20.74	0.896
31.11	1.188
41.48	1.443
Unknown	0.785

To find the confidence interval, we need a measure of the concentration of the unknown and its uncertainty. To find these, we need to fit the data and calculate the uncertainty in an x value determined from the fit. Your process should include:

- Determine the least squares line for the data
 - Perform the requisite calculations to find slope and intercept
- Use the slope and intercept to find the concentration for the unknown.
- From the calibration data, determine the standard deviation about regression (s_y)
- Use s_y and the data to find s_x , the uncertainty of an x -value calculated from the line using an equation such as:
- Once s_x is determined, the confidence interval is $value \pm ts$ where t is determined by $n-2$ degrees of freedom. In this case, our number of degrees of freedom is $(5-2 = 3)$, so the appropriate value for t is 3.182
- Note that there is no square root of n in the confidence interval calculation.

7. You have been given the task of teaching a quantitative analysis student, Al Thumbs, the proper preparation and use of a Class A buret for titrations in order to obtain high quality quantitative results. Clearly describe your instructions to this student, include reminders of common pitfalls Al should avoid.

Your discussion for should include the following:

- Procedure for cleaning the buret (and tip)
- Taking care to avoid air bubbles in the tip
- Being sure to allow time for the walls to drain and material to react before reading
- Reading the buret from the bottom of the meniscus, with the meniscus at eye level
- Estimating readings to 1/10 of the smallest graduation (0.01 mL on a 50 mL buret)
- Shoot for consistent endpoint color.
- Taking care to "cut" drops near the endpoint

Possibly Useful Information

$m = \frac{m' \left(1 - \frac{d_a}{d_w}\right)}{\left(1 - \frac{d_a}{d}\right)}$	<p>Density of air = 0.012 g/ml Density of balance weights = 8.0 g/ml</p>
$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$	$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
$e_c = \sqrt{e_A^2 + e_B^2}$	$e_c = C \sqrt{\left(\frac{e_A}{A}\right)^2 + \left(\frac{e_B}{B}\right)^2}$
$t_{\text{calculated}} = \frac{ \text{known value} - \bar{x} }{s} \sqrt{n}$	$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$
$t_{\text{calculated}} = \frac{ \bar{x}_1 - \bar{x}_2 }{s_{\text{pooled}}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$	$s_{\text{pooled}} = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$
$t_{\text{calculated}} = \frac{\bar{d}}{s_d} \sqrt{n}$	$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$
$s_x = \frac{s_y}{ m } \sqrt{\frac{1}{k} + \frac{1}{n} + \frac{(y - \bar{y})^2}{m^2 \sum (x_i - \bar{x})^2}}$	$s_y = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-2}} = \sqrt{\frac{\sum d_i^2}{n-2}}$
$s_m^2 = \frac{s_y^2 \times n}{D}$	$s_b^2 = \frac{s_y^2 \sum x_i^2}{D}$
$y_{\text{LOD}} = y_{\text{blank}} + 3s$	$F_{\text{calculated}} = \frac{(s_1)^2}{(s_2)^2}$
$Q_{\text{calculated}} = \frac{\text{gap}}{\text{range}}$	$G_{\text{calculated}} = \frac{ \text{suspect value} - \bar{x} }{s}$

Values of Student's t

Degrees of Freedom	Confidence Level (%)			
	90	95	99.5	99.9
1	6.314	12.706	127.32	636.61
2	2.920	4.303	14.089	31.598
3	2.353	3.182	7.453	12.924
4	2.132	2.776	5.598	8.610
5	2.015	2.571	4.773	6.869
6	1.943	2.447	4.317	5.959
7	1.895	2.365	4.029	5.408
8	1.860	2.306	3.832	5.041
9	1.833	2.262	3.690	4.781
10	1.812	2.228	3.581	4.587
∞	1.645	1.960	2.807	3.291

Values of Q for rejection of data

# of Observations	Q (90% Confidence)
4	0.76
5	0.64
6	0.56

Grubbs Test for Outliers

# of Observations	G _{critical} At 95% confidence
4	1.463
5	1.672
6	1.822

Critical Values of F at the 95% Confidence Level

Degrees of freedom for s ₂	Degrees of freedom for s ₁								
	2	3	4	5	6	7	8	9	10
2	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
3	9.55	9.28	9.12	9.01	8.94	8.89	8.84	8.81	8.79
4	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74

Periodic Table of the Elements

1 IA H Hydrogen 1.008	2 IIA 2A He Helium 4.003																	13 IIIA 3A B Boron 10.811	14 IVA 4A C Carbon 12.011	15 VA 5A N Nitrogen 14.007	16 VIA 6A O Oxygen 15.999	17 VIIA 7A F Fluorine 18.998	18 VIIIA 8A Ne Neon 20.180
3 Li Lithium 6.941	4 Be Beryllium 9.012																	5 Al Aluminum 26.982	6 Si Silicon 28.086	7 P Phosphorus 30.974	8 S Sulfur 32.066	9 Cl Chlorine 35.453	10 Ar Argon 39.948
11 Na Sodium 22.990	12 Mg Magnesium 24.305	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948						
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.867	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.631	33 As Arsenic 74.922	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 83.798						
37 Rb Rubidium 85.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.95	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.711	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.294						
55 Cs Cesium 132.905	56 Ba Barium 137.328	57-71 Lanthanide Series	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.227	78 Pt Platinum 195.085	79 Au Gold 196.967	80 Hg Mercury 200.592	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium (208.982)	85 At Astatine 209.987	86 Rn Radon 222.018						
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103 Actinide Series	104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (269)	109 Mt Meitnerium (278)	110 Ds Darmstadtium (281)	111 Rg Roentgenium (280)	112 Cn Copernicium (285)	113 Nh Nihonium (286)	114 Fl Flerovium (289)	115 Mc Moscovium (289)	116 Lv Livermorium (293)	117 Ts Tennessine (294)	118 Og Oganesson (294)						
57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.243	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.500	67 Ho Holmium 164.930	68 Er Erbium 167.259	69 Tm Thulium 168.934	70 Yb Ytterbium 173.055	71 Lu Lutetium 174.967									
89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium (254)	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium (262)									

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