

This take-home least-squares problem will account for 15 possible points on this exam.

You must complete the following individually. You may use your textbook and notes, but may not receive assistance from your classmates or anyone other than Dr. Lamp. This signed sheet must accompany the completed problem. Attach any computer output to this sheet and show other work in the space below. By signing below, you certify that you completed the problems in accordance with these rules. No credit will be given to unsigned papers.

Signature(s) _____ Date _____

Tooth enamel consists mainly of the mineral hydroxyapatite, $\text{Ca}_{10}(\text{PO}_4)_6(\text{OH})_2$. Trace elements in teeth of archeological specimens provide anthropologists with clues about the diet and diseases of ancient people. Below is data from an analysis of strontium in extracted wisdom teeth using the method of standard additions. Solutions were prepared with a constant total volume of 10.0 mL containing 0.750 mg dissolved tooth enamel plus various concentrations of added strontium.

Added Strontium (ppb, ng/mL)	Signal (arbitrary units)
0.00	28.0
2.50	34.3
5.00	42.8
7.50	51.5
10.00	58.6

- a. Perform a linear least-squares analysis on the data and determine the slope and intercept as well as the 95% confidence interval for both. Attach a printout of your plot and any other computer output you may have used (spreadsheet, Minitab, etc.). (10 points)

Plots are on the next page. Confidence intervals for slope and intercept are:

$$m = 3.1_3 \pm 0.3_0 \text{ ppb}^{-1}$$

$$b = 27.4 \pm 1.8$$

- b. Determine the strontium concentration in the tooth enamel in ppm ($\mu\text{g/g}$), with its 95% confidence interval. The standard deviation of the x-intercept is calculated as shown below: (5 points)

$$s_{x\text{-int}} = \frac{s_y}{m} \sqrt{\frac{1}{n} + \frac{\bar{y}^2}{m^2 \sum (x_i - \bar{x})^2}}$$

Solving for the x-intercept and its confidence interval, we find:

$$x\text{-int.} = -8.7 \pm 1.4 \text{ ppb}$$

We need to convert this to a composition of tooth enamel by determining the mass of Sr in the solution:

$$10.0 \text{ mL} \times \frac{8.7 \pm 1.4 \text{ ng Sr}}{1 \text{ mL}} = 87 \pm 14 \text{ ng Sr in the 0.750 mg tooth enamel}$$

This translates to:

$$\frac{87 \pm 14 \text{ ng Sr}}{0.750 \text{ mg}} \times \frac{116 \pm 18.7 \text{ ng Sr}}{\text{mg enamel}} = 120 \pm 20 \text{ ppm Sr in the enamel}$$

	x	y	xy	x ²	n	y _{calc}	d	d ²	(y-y _{bar}) ²	y _{Unknown}
	0.00	28.0	0	0	5	27.36	0.64	0.4096	226.2	
	2.50	34.3	85.75	6.25		35.2	-0.9	0.81	76.388	
	5.00	42.8	214	25		43.04	-0.24	0.0576	0.0576	
	7.50	51.5	386.25	56.25		50.88	0.62	0.3844	71.572	
	10.00	58.6	586	100		58.72	-0.12	0.0144	242.11	
			0	0		0	0	0		
			0	0		0	0	0		
			0	0		0	0	0		
			0	0		0	0	0		
			0	0		0	0	0		
			0	0		0	0	0		
Sums	25	215.2	1272	187.5		215.2	0	1.676	616.33	
	D =	312.5				$D = (E12 * F2) - (B12 * B12)$				
	m =	3.136				$m = ((D12 * F2) - (C12 * B12)) / C14$				
	b =	27.36				$b = ((E12 * C12) - (D12 * B12)) / C14$				
	S_y =	0.74744				$S_y = \text{SQRT}(I12 / (F2 - 2))$				
	S_m =	0.09454	% S_m =	3.0148		$S_m = \text{SQRT}((C17^2 * F2) / C14)$				
	S_b =	0.57896	% S_b =	2.1161		$S_b = \text{SQRT}((C17^2 * E12) / C14)$				
	S_x =	0.48926	% S_x =	-5.608		$S_x = (C17 / \text{ABS}(C15)) * \text{SQRT}((1/1) + (C21^2 * F2 / C14) + (E12 / C14) - ((2 * C21 * B12) / C14))$				
	x_{unk} =	-8.7245				$x_{unk} = (J2 - C16) / C15$				
	x-int. =	-8.7245				$x\text{-int.} = -C16 / C15$				
	S_{x-int.} =	0.42728	% S_{x-int.} =	-4.897		$S_{x\text{-int.}} = (C17 / C15) * \text{SQRT}((1/F2) + \text{AVERAGE}(C2:C11)^2 / (C15^2 * \text{DEVSQ}(B2:B11)))$				
	R² =	0.99728				$R^2 = 1 - (I12 / J12)$				
						$t = \text{TINV}(0.05, F2 - 2)$				

	value	unc. (s)	% rel unc.
m	3.1360	0.095	3.0148
b	27.3600	0.579	2.1161

95% Confidence Intervals			t= 3.1824
m =	3.136	+/-	0.3009
b =	27.36	+/-	1.8425
x-int. =	-8.724	+/-	1.3598

