

Complete these problems on separate paper and staple it to this sheet when you are finished. Please initial each sheet as well. Clearly mark your answers. YOU MUST SHOW YOUR WORK TO RECEIVE CREDIT.

Homework Problem (10 pts): The signal processing homework problem will be graded out of 10 points and added to the total for these problems.

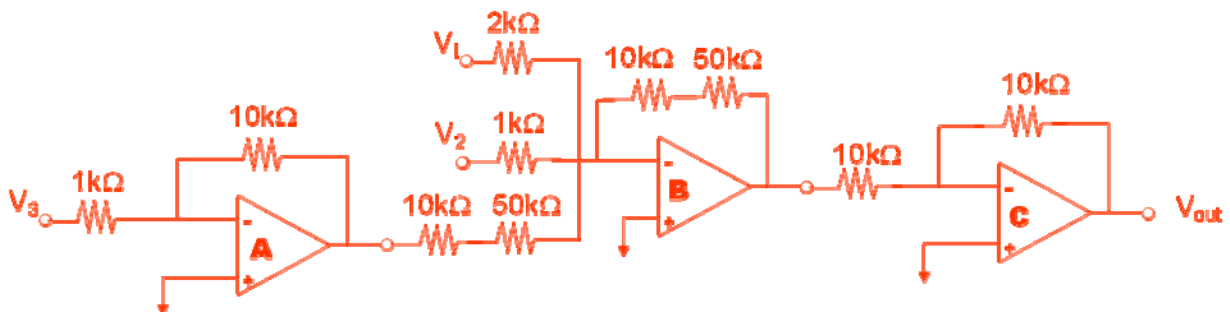
Warm-up (2 pts each)

1. The range of frequencies that a measurement is sensitive to is called the frequency bandwidth.
2. Application of large reverse-bias potentials to diodes can lead to breakdown (or Zener breakdown), resulting in a large current at essentially constant potential.
3. The linear dynamic range is the span of concentrations over which instrument response is directly proportional to concentration.
4. The sensitivity of a measurement can be described using the slope of the calibration curve for the measurement.

Answer in a few sentences, or with a calculation. Complete **FOUR** of the following. Clearly indicate which problem is not to be graded. Show all work for calculations. (10 pts each)

5. You have a box that contains 50 op-amps and ten each of the following resistors: 1, 2, 5, 10, 20, 50 kΩ. Design a circuit to do the following calculation: $v_{out} = 30(v_1 + 2v_2) - 10v_3$ (You may not need all of the components in your box.). You do not need to do calculations to prove your result; it just needs to be correct.

There are several options that will do the trick. You need to be sure that your circuit performs the function $v_{out} = 30v_1 + 60v_2 - 10v_3$ and that the sign of the output is correct. The circuit below is an example:



The opamp **A** in the circuit produces $v_A = -10v_3$. Opamp **B** produces $v_B = -(30v_1 + 60v_2 + v_A)$, and opamp **C** produces $v_{out} = v_C = -v_B = 30v_1 + 60v_2 - 10 v_3$.

6. Considering the four types of noise we discussed, what characteristics should a measurement have to minimize the contribution of all four noise sources?

The measurement should have narrow bandwidth to minimize shot and thermal noise, operate at high frequency to minimize flicker noise, and operate at a frequency away from environmental noise sources. You may also mention a desire to have a low resistance system and low temperature.

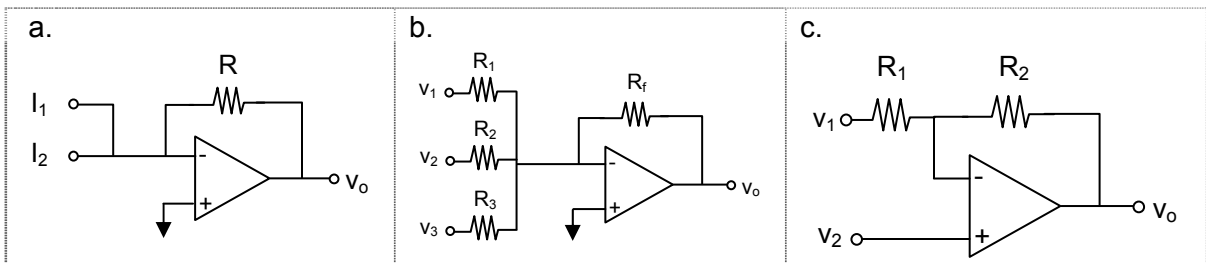
7. Clearly outline the steps you would take to determine the detection limit for an absorbance measurement based around Beer's Law ($A = abc$). Identify the data that you would need to collect and how you would use the data to calculate a LOD.

The detection limit is the concentration that corresponds to a signal that is three (or two) times the standard deviation of the background above the background. In order to determine the detection limit, you need multiple measurements of the blank to define the background and its standard deviation. With these values, you can calculate the signal at the detection limit as $\text{Signal}_{\text{DL}} = \text{Signal}_{\text{blank}} + 3s_{\text{blank}}$. You must then convert this signal into a concentration. This can be done by preparing a calibration curve by measuring the absorbance at several known concentrations and using the Beer's law relationship to convert the $\text{Signal}_{\text{DL}}$ (which is an absorbance) into its corresponding concentration.

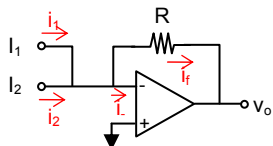
8. There are the two primary properties of an A/D or D/A converter that limit its ability to accurately convert between analog and digital values. Identify each of these parameters and describe how they limit the accuracy of a conversion.

The two parameters are the number of bits and the size of the input (or output) range of the device. The combination of these parameters determines the size of the least significant bit, which is the smallest change in signal that the device can accommodate. The larger the number of bits and the closer the range of the device is to the size of the signal input (or output), the better the conversion will be.

9. Derive the relationship between the input and output signals for **two** of the following circuits. Show all work.



a.



$i_1 + i_2 = i_f + i_-$, but $i_- = 0$, so;

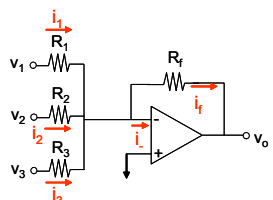
$$i_1 + i_2 = \frac{v_- - v_o}{R_2}$$

But, $v_- = v_+ = 0$

$$i_1 + i_2 = \frac{-v_o}{R_2}$$

$$v_o = -R_2(i_1 + i_2)$$

b.



$i_1 + i_2 + i_3 = i_f + i_-$, but $i_- = 0$, so;

$$\frac{v_1 - v_-}{R_1} + \frac{v_2 - v_-}{R_2} + \frac{v_3 - v_-}{R_3} = \frac{v_- - v_o}{R_f}$$

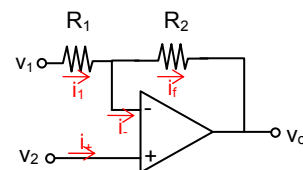
But, $v_- = v_+ = 0$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = \frac{-v_o}{R_f}$$

and

$$\frac{v_1 R_f}{R_1} + \frac{v_2 R_f}{R_2} + \frac{v_3 R_f}{R_3} = -v_o$$

c.



$i_1 = i_2 + i_-$, but $i_- = 0$, so;

$$\frac{v_1 - v_-}{R_1} = \frac{v_- - v_o}{R_2}$$

But, $v_- = v_+ = v_2$

$$\frac{v_1 - v_2}{R_1} = \frac{v_2 - v_o}{R_2}$$

$$v_2 - \frac{R_2(v_1 - v_2)}{R_1} = v_o$$

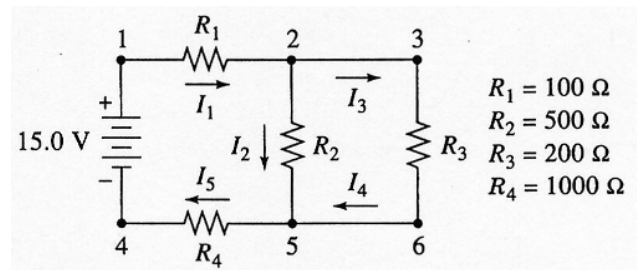
or

$$v_o = v_2(1 + (R_1/R_2)) - (R_1/R_2)v_1$$

A little more involved. Complete three of the following. Be clear and concise. Clearly indicate which problem is not to be graded. (14 pts each)

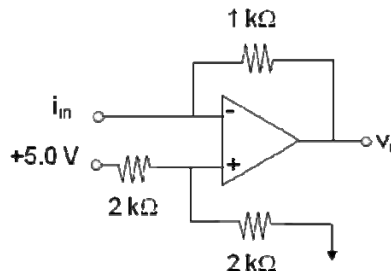
10. For the following circuit, the current, I_4 is 8.65 mA. Calculate:

- the potential difference across resistor R_2 .
- the current through resistor R_4 .
- the power dissipated by resistor R_1 .
- the potential difference between points 3 and 4.



- Since they are in parallel, the potential across resistor R_2 must be the same as the potential across resistor R_3 . This potential is $V = I_4 R_3 = (8.65 \text{ mA})(200 \Omega) = \mathbf{1.73 \text{ V}}$
- If we can find I_5 , we can calculate the potential across R_4 . We know that $I_5 = I_2 + I_4$, so:
 $I_5 = V_2/R_2 + 8.65 \text{ mA} = (1.73 \text{ V}/500 \Omega) + 8.65 \text{ mA} = 3.46 \text{ mA} + 8.65 \text{ mA} = \mathbf{12.11 \text{ mA}}$
 So, $V_4 = I_5 R_4 = (12.11 \text{ mA})(1000 \Omega) = \mathbf{12.11 \text{ V}}$
- $P = I^2 R = (12.11 \times 10^{-3} \text{ A})(100 \Omega) = \mathbf{0.147 \text{ W}}$
- The potential between points 3 and 4 corresponds to the potential sum of the potential between points 3 and 6 and the potential between points 4 and 6. The potential between points 3 and 6 is that potential across resistor R_3 , which we calculated to be 1.73 V. The potential between points 4 and 6 corresponds to the potential across R_4 , which is $i_5 R_4 = 12.11 \text{ mA} \times 1000 \Omega = 12.11 \text{ V}$. Therefore the potential between points 3 and 4 is $1.73 \text{ V} + 12.11 \text{ V} = \mathbf{13.84 \text{ V}}$

11. Given the circuit below, what output would you expect for a 1.25 mA input current? Derive the relationship between v_o and i_{in} . How would you redesign this circuit to make it suitable for measurement of low current signals (micro- to nano-amp signals)?



$$\frac{5-v_+}{2 \text{ k}\Omega} = \frac{v_+-0}{2 \text{ k}\Omega}$$

$$v_+ = v_- = 2.5 \text{ V}$$

$$i_{in} = i_o + i_- = i_o$$

$$i_{in} = \frac{v_--v_o}{1 \text{ k}\Omega} = \frac{2.5 \text{ V}-v_o}{1 \text{ k}\Omega}$$

$$\mathbf{2.5 \text{ V} - (1 \text{ k}\Omega)i_{in} = v_o = 1.25 \text{ V}}$$

Sensitivity can be increased by increasing the gain by increasing the value of the feedback resistor and by decreasing the +5.0 V reference voltage (remember lab?).

12. Ensemble averaging (also known as *coadding*) and boxcar averaging are two common software-based approaches to improving signal to noise ratios (S/N). Concisely describe each approach and a scenario where boxcar averaging would be preferred over ensemble averaging for S/N improvement. What pitfalls does one need to be concerned with for each method?

You should discuss how each method works and its limitations. Here are some items I'd expect to see.

Ensemble Averaging: Several datasets are collected and "averaged" so that the non-random analytical signal is reinforced while the random noise is allowed to cancel. You would expect the S/N ratio to increase as the square root of the number of scans averaged increases.

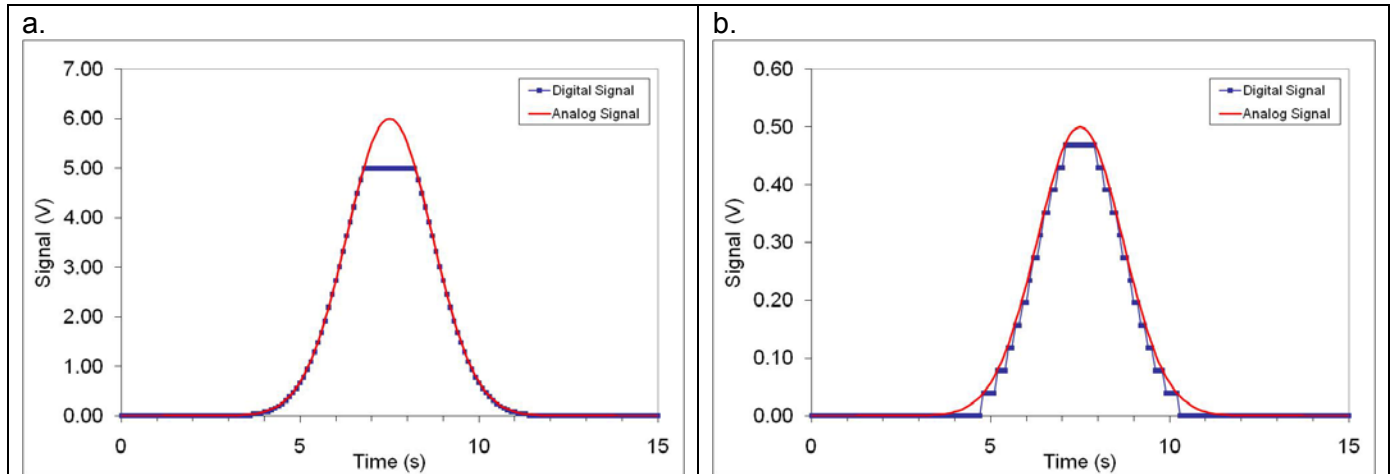
Challenges: as # of scans increases, the benefit diminishes so there is a point where the return is not worth the investment in time required to do more scans. Also, collecting more scans requires that the measurement (instrument and sample) be stable for a long period of time.

Boxcar Averaging: Adjacent data points (typically 3, 5, or 7) within a single dataset are averaged to and replaced by a single data point. This averaging should serve to emphasize the lower frequency characteristics of the signal while de-emphasizing the high frequency noise. Boxcar averaging works on a single dataset, but you need to be careful so that you have enough data that when the boxcar is run, the result is still representative of the real data.

In order for ensemble averaging to occur, you must be able to collect several sets of data, where noise is the only component that is varying. Any scenario that prevents this will prevent the use of ensemble averaging. Boxcar averaging is useful when you only have a single dataset to work with. Such scenarios would include:

- Destructive measurements and limited amounts of sample
- Time consuming analysis
- Instrument instability and drift
- Transient signals

13. The plots below show the digital representation of two Gaussian analog signals, as acquired by an eight bit analog to digital converter (ADC) with an input range of ± 5.0 V. In neither case is the conversion ideal. For each case, explain what has led to the non-ideal conversion and suggest a solution to each problem that does not involve buying a new ADC.



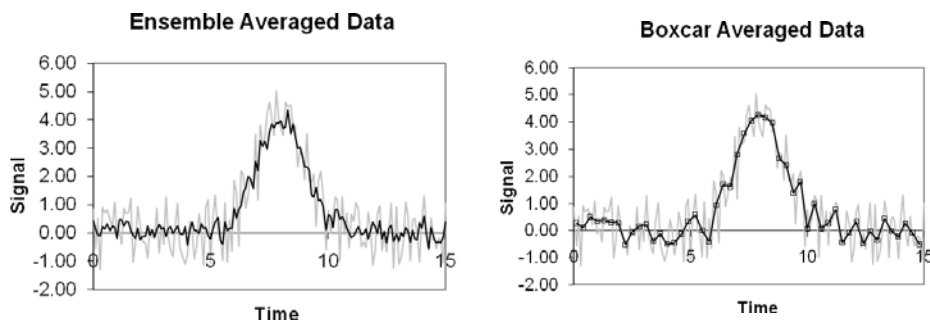
For dataset A, the magnitude of the analog signal (6V) is outside the input range of the ADC. Therefore any signal above 5V is recorded as 5V, causing the peak to be “clipped”. To resolve this problem, the analytical signal must be attenuated (de-amplified) so that it no longer overloads the ADC. Solutions to both of these situations could be accomplished using a simple operational amplifier circuit.

For dataset B the “jagged” nature of the data is an indication that the acquisition resolution is being limited by the size of the least significant bit. This is called “bit noise” or a “bit limited” situation. This arises because the size of the analytical signal is small relative to the size of the least significant bit for the acquisition system (in this case $10\text{V}/2^8$ or 39 mV). One possible solution would be to amplify the analog signal to allow it to fill up more of the input range of the ADC, thus making the impact of the size of the LSB minimal.

“Homework” Problem

To determine the S/N ratio, you must estimate the noise in the data and the magnitude of the signal (at the peak). For all three situations, a reasonable estimate of the magnitude of the signal is 4 units (the size of the peak in the “clean” data). To estimate the noise, determine the standard deviation of the background or baseline. I chose to determine the standard deviation of the data from time zero to three.

- a) Single noisy dataset: $S = 4.0$, $N = 0.68$, $S/N = 5.9$
- b) Ensemble average: $S = 4.0$, $N = 0.22$, $S/N = 18.2$. This is an improvement of about a factor of three, which is a little larger than the predicted $(8)^{1/2}$ improvement, but reasonable.
- c) Boxcar average: $S = 4.0$, $N = 0.27$, $S/N = 14.8$. This is also an improvement, although we don’t have a “target” level to compare to.



Possibly Useful Information

$\frac{dQ}{dt} = I = C \frac{dV}{dt}$	$P = IV = I^2R$
$V = IR$	$v_o = A(v_+ - v_-)$
$v_{rms} = \sqrt{4kTR\Delta f}$	$001 + 001 = 010$
$\Delta f = \frac{1}{3t_r}$	$i_{rms} = \sqrt{2ie\Delta f}$
$S_m = \bar{S}_{bl} + ks_{bl}$	$c_m = \frac{S_m - \bar{S}_{bl}}{m}$

